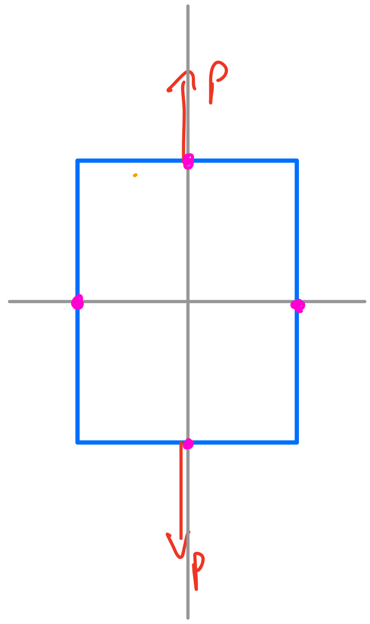
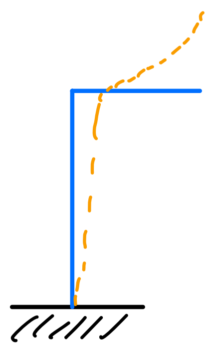
1. MENEBREA pour les hyperstatiques
2. CASTIGLIANO pour les déplacements

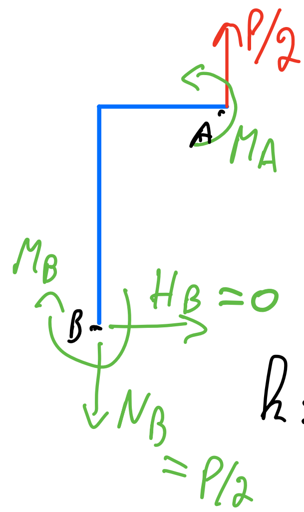
(rappel : petites déformations)



2 axes de symétrie on étudie 1/4 du système

$$k = 3 - 2 = 1$$





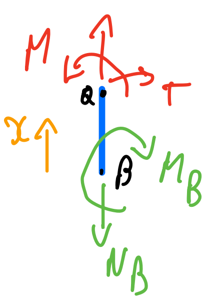
$$k = 3 + 1 - 3 = 1$$

Calculer $M_f(x)$ pour :

$$\frac{\partial V}{\partial M_B} = 0 \quad - \quad 0 = 4 \cdot \frac{1}{EI} \int_A^B M(x) \frac{\partial M(x)}{\partial M_B} dx \qquad \text{MENABREA} \quad (1)$$

$$\delta_A = 4 \frac{1}{EI} \int_A^B M(x) \frac{\partial M(x)}{\partial P} dx \qquad \text{CASTIGLIANO} \quad (2)$$

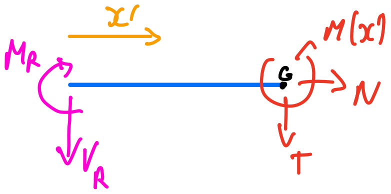
PARTIE VERTICALE



$$\Sigma M_{Q_z} = 0 \qquad M_B - M(x) = 0 \qquad M(x) = M_B$$

$$\frac{\partial M}{\partial M_B} = 1 \qquad \frac{\partial M}{\partial P} = 0$$

PARTIE HORIZONTALE



$$M_R = M(x=b) = M_B$$
$$V_R = N(x=b) = P/2$$

$$\sum M_{z_G} = 0 \qquad M(x') - M_R + V_R\, x' = 0$$
$$M(x') = M_B - \frac{P}{2}x' \qquad \frac{\partial M}{\partial M_B} = 1 \qquad \frac{\partial M}{\partial P} = \frac{-x'}{2}$$

eq 1 : $\dfrac{\partial U}{\partial M_B} = 0$

$$0 = \int_{x=0}^{b} M_B \cdot 1\, dx + \int_{x'=0}^{a} \left(M_B - \frac{P}{2}x'\right)\cdot 1\, dx'$$

$$0 = M_B\, b + M_B\, a - \frac{P}{4}a^2$$

$$\boxed{M_B = \frac{Pa^2}{4(a+b)}}$$

eq 2 $\quad \delta_A = \dfrac{\partial U}{\partial P} = 4\, \dfrac{\partial U_{quart}}{\partial P}$

$$= \frac{4}{EI}\int_{x=0}^{b} M_B \cdot \underbrace{\cancel{0}}_{0}\, dx + \frac{4}{EI}\int_{x'=0}^{a} \left(M_B - \frac{P}{2}x'\right)\left(-\frac{x'}{2}\right) dx'$$

$$= \frac{-4}{EI}\left(M_B \frac{a^2}{4} - \frac{P a^3}{12}\right) = \frac{Pa^3}{12\, EI}\,\frac{a+4b}{a+b}$$

$$\boxed{\delta_A = \frac{Pa^3}{12\, EI}\left(\frac{a+4b}{a+b}\right)}$$

forte dépendance en a comparé à b